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Present and Future Value of Annuities

By [Shauna Carther](#) | January 24, 2018 — 9:15 AM EST

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At some point in your life, you may have had to make a series of fixed payments over a period of time – such as rent or car payments – or have received a series of payments over a period of time, such as interest from [bonds](#) or [CDs](#). These are called annuities (a more generic use of the word – not to be confused with the specific financial product called an annuity, though the two are related). If you [understand the time value of money](#), you're ready to learn about annuities and how their present and future values are calculated.

Annuities are essentially a series of fixed payments required from you, or paid to you, at a specified frequency over the course of a fixed time period. Payment frequencies can be yearly, semi-annually (twice a year), quarterly and monthly. There are two basic types of annuities: ordinary annuities and annuities due.

- **Ordinary annuity:** Payments are required at the end of each period. For example, straight bonds usually make [coupon](#) payments at the end of every six months until the bond's maturity date.
- **Annuity due:** Payments are required at the beginning of each period. Rent is an example of annuity due. You are usually required to pay rent when you first move in at the beginning of the month, and then on the first of each month thereafter.

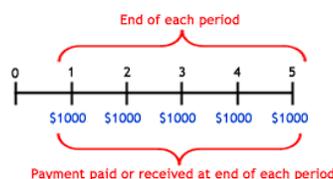
Since the present and future value calculations for ordinary annuities and annuities due are slightly different, we will discuss them separately.

ORDINARY ANNUITIES

Calculating the Future Value

If you know how much you can invest per period for a certain time period, the future value (FV) of an ordinary annuity formula is useful for finding out how much you would have in the future. If you are making payments on a [loan](#), the future value is useful in determining the total cost of the loan. If you know how much you plan to invest each year and the [fixed rate of return](#) your annuity guarantees – or, for loans, the amount of your payments and the given interest rate – you can easily determine the value of your account at any point in the future.

Let's now run through Example 1. Consider the following annuity [cash flow](#) schedule:



To calculate the [future value of the annuity](#), we have to calculate the future value of each cash flow. Let's assume that you are receiving \$1,000 every year for the next five years, and you invest each payment at 5% interest. The following diagram shows how much you would have at the end of the five-year period:

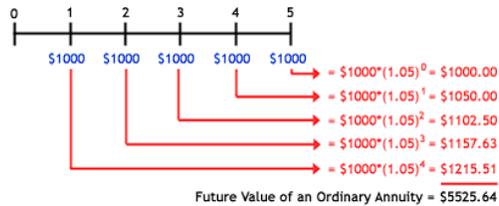
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Since we have to add the future value of each payment, you may have noticed that if you have an ordinary annuity with many cash flows, it would take a long time to calculate all the future values and then add them together. Fortunately, mathematics provides a formula that serves as a shortcut for finding the **accumulated value** of all cash flows received from an ordinary annuity:

$$FV_{\text{Ordinary Annuity}} = C * \left[\frac{(1+i)^n - 1}{i} \right]$$

where c = cash flow per period

i = interest rate

n = number of payments

Using the above formula for Example 1 above, this is the result:

$$FV_{\text{Ordinary Annuity}} = \$1000 * \left[\frac{(1+0.05)^5 - 1}{0.05} \right] = \$1000 \times [5.53] = \$5525.63$$

Note that the one-cent difference between \$5,525.64 and \$5,525.63 is due to a **rounding error** in the first calculation. Each value of the first calculation must be rounded to the nearest penny – the more you have to round numbers in a calculation, the more likely rounding errors will occur. So, the above formula not only provides a shortcut to finding the FV of an ordinary annuity, but also gives a more accurate result.

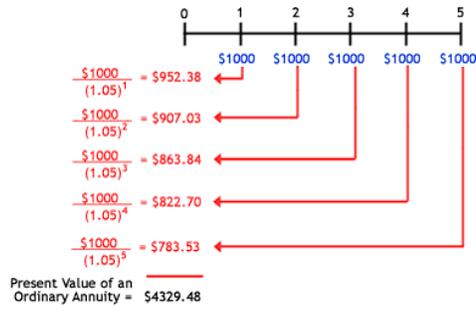
Calculating the Present Value

The **present value of an annuity** is simply the current value of all the income generated by that investment in the future. This calculation is predicated on the concept of the **time value of money**, which states that a dollar now is worth more than a dollar earned in the future. Because of this, present value calculations use the number of time periods over which income is generated to discount the value of future payments.

If you would like to determine today's value of a future payment series, you need to use the formula that calculates the present value (PV) of an ordinary annuity. This is the formula you would use as part of a **bond pricing calculation**. The PV of an ordinary annuity calculates the present value of the coupon payments that you will receive in the future.

For Example 2, we'll use the same annuity cash flow schedule as we did in Example 1. To obtain the total **discounted** value, we need to take the present value of each future payment and, as we did in Example 1, add the cash flows together.

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Again, calculating and adding all these values will take a considerable amount of time, especially if we expect many future payments. Though numerous [online calculators](#) can determine the present value of an annuity, the formula for a regular annuity is not overly complicated to calculate by hand, if we use a mathematical shortcut for PV of an ordinary annuity.

$$PV_{\text{Ordinary Annuity}} = C * \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

where c = cash flow per period

i = interest rate

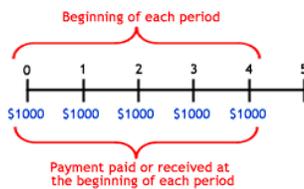
n = number of payments

The formula provides us with the PV in a few easy steps. Here is the calculation of the annuity represented in the diagram for Example 2:

$$PV_{\text{Ordinary Annuity}} = \$1000 * \left[\frac{1 - (1 + 0.05)^{-5}}{0.05} \right] = \$1000 * [4.33] = \$4329.48$$

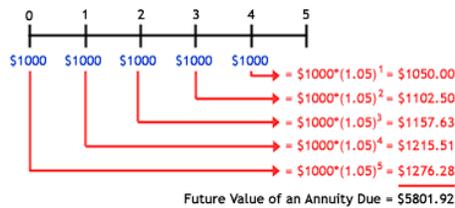
Calculating the Future Value

When you are receiving or paying cash flows for an annuity due, your cash flow schedule would appear as follows:



Since each payment in the series is made one period sooner, we need to discount the formula one period back. A slight modification to the FV-of-an-ordinary-annuity formula accounts for payments occurring at the beginning of each period. In Example 3, let's illustrate why this modification is needed when each \$1,000 payment is made at the beginning of the period rather than at the end (interest rate is still 5%):

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Notice that when payments are made at the beginning of the period, each amount is held longer at the end of the period. For example, if the \$1,000 was invested on January 1 rather than December 31 each year, the last payment before we value our investment at the end of five years (on December 31) would have been made a year prior (January 1) rather than the same day on which it is valued. The future value of annuity formula would then read:

$$FV_{\text{Annuity Due}} = C * \left[\frac{(1+i)^n - 1}{i} \right] * (1+i)$$

where c = cash flow per period

i = interest rate

n = number of payments

Therefore,

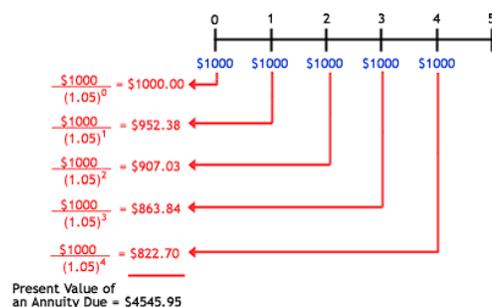
$$FV_{\text{Annuity Due}} = \$1000 * \left[\frac{(1+0.05)^5 - 1}{0.05} \right] * (1+0.05) = \$1000 * 5.53 * 1.05 = \$5801.91$$

ANNUITY DUE

Calculating the Present Value

For the present value of an annuity-due formula, we need to discount the formula one period forward as the payments are held for a shorter amount of time. When calculating the present value, we assume that the first payment was made today.

We could use this formula for calculating the present value of your future rent payments as specified in a [lease](#) you sign with your [landlord](#). Let's say for Example 4 that you make your first rent payment at the beginning of the month and are evaluating the present value of your five-month lease on that same day. Your present value calculation would work as follows:



Of course, we can use a formula shortcut to calculate the present value of an annuity due:

$$PV_{\text{Annuity Due}} = C * \left[\frac{1 - (1+i)^{-n}}{i} \right] * (1+i)$$

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where c = cash flow per period

i = interest rate

n = number of payments

Therefore,

$$PV_{\text{Annuity Due}} = \$1000 * \left[\frac{1 - (1 + 0.05)^{-5}}{0.05} \right] * (1 + 0.05) = \$1000 * 4.33 * 1.05 = \$4545.95$$

Recall that the present value of an ordinary annuity returned a value of \$4,329.48. The present value of an ordinary annuity is less than that of an annuity due because the further back we discount a future payment, the lower its present value – each payment or cash flow in an ordinary annuity occurs one period further into the future.

The Time Value of Money

The future value calculation is based on the concept of the time value of money. This simply means a dollar earned today is worth more than a dollar earned tomorrow, because funds you control now can be invested and earn interest over time. Therefore, the future value of an annuity is greater than the sum of all your **investments** because those contributions have been earning interest over time. For example, the future value of \$1,000 invested today at 10% interest is \$1,100 one year from now. A single dollar today is worth \$1.10 in a year because of the time value of money.

Assume you make annual payments of \$5,000 to your ordinary annuity for 15 years. It earns 9% interest, **compounded** annually.

$$FV = \$5,000 * \{((1 + 0.09)^{15}) - 1\} \div 0.09$$

$$= \$5,000 * \{((1.09^{15}) - 1) \div 0.09\}$$

$$= \$5,000 * 2.642 \div 0.09$$

$$= \$5,000 * \$146,804.58$$

Without the power of interest compounding, your series of \$5,000 contributions is only worth \$75,000 at the end of 15 years. Instead, with **compound interest**, the future value of your annuity is almost twice that at \$146,804.58.

To calculate the future value of an annuity due, simply multiply the ordinary future value by $1 + i$ (the interest rate). In the above example, the future value of annuity due with the same parameters is simply $\$146,804.58 * (1 + 0.09)$, or \$160,016.99.

Present Value Considerations

When calculating the present value of an annuity, it is important that all variables are consistent. If the annuity generates annual payments, for example, the interest rate must also be expressed as an annual rate. If the annuity generates monthly payments, for example, the interest rate must also be expressed as a monthly rate.

Assume an annuity has a 10% interest rate that generates annual payments of \$3,000 for the next 15 years. The present value of this annuity is:

$$= \$3,000 * (((1 - (1 + 0.1)^{-15}) \div 0.1)$$

$$= \$3,000 * ((1 - .239392) \div 0.1)$$

$$= \$3,000 * (0.760608 \div 0.1)$$

$$= \$3,000 * 7.60608$$

$$= \$22,818$$

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if the annuity paid monthly installments of \$250 instead of annual payments of \$3,000, the interest rate is $10\% \div 12$, or 0.833%, and the number of periods is 15×12 , or 180.

The Bottom Line

Now you can see how annuities affect how you calculate the present and future value of any amount of money. Remember that the payment frequencies, or number of payments, and the time at which these payments are made (whether at the beginning or end of each payment period) are all variables you need to account for in your calculations.

When planning for retirement, it is important to have a good idea of how much income you can rely on each year. While it may be relatively easy to keep track of how much you put into [employer-sponsored retirement plans](#), [individual retirement accounts \(IRAs\)](#) and annuities, it is not always so easy to know how much you will get out. Luckily, when it comes to fixed-rate annuities or plans invested in [fixed-rate securities](#), there is a simple way to calculate how much money you can expect to have available after retirement based on how much you put into the account during your working years.

See also [For what types of financial instruments would I want to calculate the present value of an annuity?](#)

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