Radius of curvature (applications)

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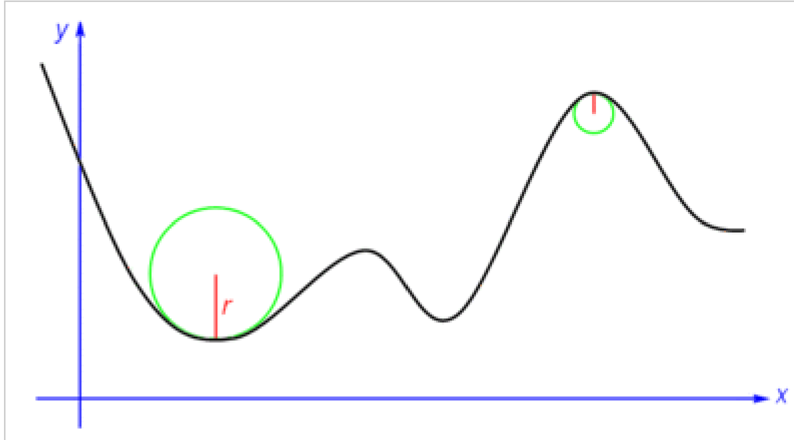
The distance from the center of a circle or sphere to its surface is its radius. For other curved lines or surfaces, the **radius of curvature** at a given point is the radius of a circle that mathematically best fits the curve at that point. In the case of a surface, the radius of curvature is the radius of a circle that best fits a *normal section*, the intersection of the surface with a plane containing the normal to the surface at a particular point.



Radius of curvature(r)

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Explanation

Imagine driving a car on a curvy road on a completely flat surface. At any one point along the way, lock the steering wheel in its position, so that the car thereafter follows a perfect circle. The car will, of course, deviate from the road, unless the road is also a perfect circle. The radius of that circle the car makes is the radius of curvature of the curvy road at the point at which the steering wheel was locked. The more sharply curved the road is at the point you locked the steering wheel, the smaller the radius of curvature.

Formula

If $\gamma : \mathbb{R} \longrightarrow \mathbb{R}^n$ is a parameterized curve in $\mathbb{R}^n$ then the radius of curvature at each point of the curve, $\rho : \mathbb{R} \longrightarrow \mathbb{R}$, is given by

$$\rho = \frac{|\gamma'|^3}{\sqrt{|\gamma'|^2\,|\gamma''|^2 - (\gamma' \cdot \gamma'')^2}}.$$

As a special case, if *f*(*t*) is a function from $\mathbb{R}$ to $\mathbb{R}$, then the curvature of its graph, $\gamma(t) = (t, f(t))$, is

$$\rho(t) = \frac{|1 + f'^2(t)|^{3/2}}{|f''(t)|}.$$

Derivation

Let γ be as above, and fix t. We want to find the radius ρ of a parameterized circle which matches γ in its zeroth, first, and second derivatives at t. Clearly the radius will not depend on the position ($\gamma(t)$), only on the velocity ($\gamma'(t)$) and acceleration ($\gamma''(t)$). There are only three independent scalars that can be obtained from two vectors *v* and *w*, namely *v*·*v*, *v*·*w*, and *w*·*w*. Thus the radius of curvature must be a function of the three scalars $|\gamma'^2(t)|$, $|\gamma''^2(t)|$ and $\gamma'(t) \cdot \gamma''(t)$.

The general equation for a parameterized circle in $\mathbb{R}^n$ is

$$g(u) = A\cos(h(u)) + B\sin(h(u)) + C$$

where $C \in \mathbb{R}^n$ is the center of the circle (irrelevant since it disappears in the derivatives), $A, B \in \mathbb{R}^n$ are perpendicular vectors of length ρ (that is, $A \cdot A = B \cdot B = \rho^2$ and $A \cdot B = 0$), and $h : \mathbb{R} \to \mathbb{R}$ is an arbitrary function which is twice differentiable at *t*.

The relevant derivatives of *g* work out to be

$$\begin{array}{lcl} |g'|^2 & = & \rho^2 (h')^2 \\ g' \cdot g'' & = & \rho^2 h' h'' \\ |g''|^2 & = & \rho^2 \left((h')^4 + (h'')^2 \right) \end{array}$$

If we now equate these derivatives of *g* to the corresponding derivatives of γ at *t* we obtain

$$\begin{array}{lcl} |\gamma'^2(t)| & = & \rho^2 h'^2(t) \\ \gamma'(t) \cdot \gamma''(t) & = & \rho^2 h'(t) h''(t) \\ |\gamma''^2(t)| & = & \rho^2 (h'^4(t) + h''^2(t)) \end{array}$$

These three equations in three unknowns (ρ, $h'(t)$ and $h''(t)$) can be solved for ρ, giving the formula for the radius of curvature:

$$\rho(t) = \frac{|\gamma'^3(t)|}{\sqrt{|\gamma'^2(t)|\,|\gamma''^2(t)| - (\gamma'(t) \cdot \gamma''(t))^2}}$$

or, omitting the parameter (*t*) for readability,

$$\rho = \frac{|\gamma'|^3}{\sqrt{|\gamma'|^2\,|\gamma''|^2 - (\gamma' \cdot \gamma'')^2}}.$$

Applications and examples

- For the use in differential geometry, see Cesàro equation.

- For the radius of curvature of the earth (approximated by an oblate ellipsoid), see Radius of curvature of the earth.
- Radius of curvature is also used in a three part equation for bending of beams.

Radius of curvature applied to measurements of the stress in the semiconductor structures

Stress in the semiconductor structure involving evaproated thin films usually results from the thermal expansion (thermal stress) during the manufacturing process. Thermal stress occurs because film depositions are usually made above room temperature. Upon cooling from the deposition temperature to room temperature, the difference in the thermal expansion coefficients of the substrate and the film cause thermal stress.[1]

Intrinsic stress results from the microstructure created in the film as atoms are deposited on the substrate. Tensile stress results from microvoids in the thin film, because of the attractive interaction of atoms across the voids.

The stress in thin film semiconductor structures results in the buckling of the wafers. The radius of the curvature of the stressed structure is related to stress tensor in the structure, and can be described by modified Stoney formula.[2] The topography of the stressed structure including radii of curvature can be measured using optical scanner methods. The modern scanner tools have capability to measure full topography of the substrate and to measure both principal radii of curvature, while providing the accuracy of the order of 0.1% for radii of curvature of 90 m and more.[3]

References:

1. ^ http://flipchips.com/tutorial/process/controlling-stress-in-thin-films/ and references within
2. ^ http://www.qucosa.de/fileadmin/data/qucosa/documents/5126/data/Stoney.pdf
3. ^ http://www.zebraoptical.com/ModelX.html

See also

- AFM probe
- Base curve radius
- Bend radius
- Curve
- Curvature
- Degree of curvature, civil engineering
- Diameter
- Minimum railway curve radius
- Radius of curvature (optics)
- Reverse curve
- Track transition curve
- Transition curve

External links

- The Geometry Center: Principal Curvatures (http://www.geom.uiuc.edu/zoo/diffgeom/surfspace/concepts/curvatures/prin-curv.html)
- 15.3 Curvature and Radius of Curvature (http://math.mit.edu/classes/18.013A/HTML/chapter15/section03.html)
- Weisstein, Eric W., "Principal Curvatures (http://mathworld.wolfram.com/PrincipalCurvatures.html)",

MathWorld.

- Weisstein, Eric W., "Principal Radius of Curvature (http://mathworld.wolfram.com/PrincipalRadiusofCurvature.html)", *MathWorld*.

Categories: Differential geometry | Curvature (mathematics)
